

Mathematics  
EXAMPLE TASKS FROM EXAMS

Michał Bernardelli

**Exercise 1.**

Find the domain and the asymptotes of the function

$$f(x) = \frac{\ln|x|}{x-1}.$$

**Exercise 2.**

Calculate the integral:

$$\int x^5 e^{-x^3} dx.$$

**Exercise 3.**

Check if the inequality is true:

$$\int_{e^2}^{+\infty} \frac{3}{x (\ln \sqrt{x})^5} dx < \sqrt{2}.$$

**Exercise 4.**

Calculate the area of the field limited by the parabola  $y = x^2$ , tangent line to this parabola at the point  $(1, 1)$  and the line  $y = 6x + 7$ . Points  $(x, y)$  of the area fulfill the condition  $x \in \langle -2; 1 \rangle$ .

**Exercise 5.**

Find the minimum and maximum value of the function

$$f(x) = \frac{1}{4}x^4 - 2x^2 + 3$$

in the interval  $\langle -1, 4 \rangle$ . Determine the points at which the function is convex.

**Exercise 6.**

Check the continuity and find the asymptotes to the graph of the function

$$f(x) = \begin{cases} \frac{1}{x} & x > 0 \\ x \sin x & x \leq 0 \end{cases}.$$

**Exercise 7.**

Calculate the integrals:

$$\text{a) } \int 2x^4 \ln x^5 dx \qquad \text{b) } \int_0^{\infty} \frac{(3+e^{-x})^2}{e^x} dx$$

**Exercise 8.**

Calculate the integral:

$$\int_0^{\infty} e^{-x} dx.$$

Calculate the area of the field lying in the first quadrant of the coordinate system limited by the curve  $y = e^{-x}$  and tangent line to this curve at the point  $(1, \frac{1}{e})$ .

**Exercise 9.**

Sketch the function

$$f(x) = |4 - (x + 2)^2|.$$

Calculate  $f(A)$  and  $f^{-1}(B)$  for  $A = \langle -3, -2 \rangle$  and  $B = \langle 3, 4 \rangle$ .

**Exercise 10.**

The area

$$D = \{(x, y) \in \mathbb{R}^2 : x \in \langle 0; 6 \rangle, y \geq (x - 3)^2, y \leq 9\}$$

was divided by the curve  $y = x^2$  into two parts  $F_1$  and  $F_2$ . Calculate the ratio of those fields.

**Exercise 11.**

Find the local extrema of the function

$$f(x) = \frac{1}{5}x^5 - \frac{5}{3}x^3 + 4x.$$

Determine the points at which the function is increasing and concave up. Find the minimum and the maximum value of the function in the interval  $\langle -1, 3 \rangle$ .

**Exercise 12.**

Calculate the integrals:

$$\text{a) } \int_{\sqrt{3}}^2 x\sqrt{x^2 - 3} dx$$

$$\text{b) } \int_1^e x^2 \ln x dx$$

**Exercise 13.**

Solve the system of linear equations

$$\begin{cases} x + 2y - z = 3 \\ 3x + y - 2z = 5 \\ 2x - y + 3z = 2 \end{cases}.$$

**Exercise 14.**

Give the partial fraction expansions for the expression:

$$\frac{1}{(x - 2)^2(x^2 + 3)(x^2 + 5)^3(x + 7)}.$$

**Exercise 15.**

Calculate the integral

$$\int \frac{x^8 - 2x^4 + 3x^3 - 9x^2 + 4}{x^5 - 5x^3 + 4x} dx.$$